Solving Linear Systems by Elimination

Remember:

Two lines that intersect at exactly one point have 1 solution(s).

Two parallel lines have no solution(s).

"Coincidental" lines have infinite solution(s).

Is this a solution to the problem? Yes or No?

1. $2x + y = 3$ T (0, 3)
   $x - 2y = -1$ $0 - 2(3) = -1$ $-6 \neq -1$ F No

2. $x - 3y = 6$ (-6, -4)
   $2x - y = -8$

3. $x - y = 3$ (4 , 1)
   $3x - y = 11$

We can solve linear systems of equations by graphing or algebraically by using the elimination process.

How to use elimination:

1. Arrange the equations with like terms in columns.
2. Multiply one or both of the equations by a number to obtain coefficients that are opposites for one of the variables.
3. Add the equations from Step 2. Combining like terms will eliminate one variable. Solve for the remaining variable.
4. Substitute the value obtained in Step 3 into either of the original equations and solve for the other variable.
5. Check the solution in each of the original equations.

Solve the linear system using elimination.

$$4x + \cancel{3y} = 16$$
$$+ \quad 2x - \cancel{3y} = 8$$
$$\frac{6x}{6} = \frac{24}{6}$$
$$x = 4$$

$$4(4) + 3y = 16$$
$$\underset{-16}{16} + 3y = \underset{-16}{16}$$
$$\frac{3y}{3} = \frac{0}{3}$$
$$y = 0$$

(4, 0)

Check: $4(4) + 3(0) = 16$
$16 = 16$
T
$2(4) - 3(0) = 8$
$8 = 8$
T

Ex 2.

$$\begin{aligned} -\cancel{x} + 2y &= -8 \\ + \quad \cancel{x} + 6y &= -16 \end{aligned}$$

$$\frac{8y}{8} = \frac{-24}{8}$$

$$y = -3$$

$$x + 6(-3) = -16$$

$$x - 18 = -16$$
$$+18 \quad +18$$
$$x = 2$$

$(2, -3)$

Check:

$-2 + 2(-3) = -8$
$-2 - 6 = -8$
$-8 = -8 \quad T$

$2 + 6(-3) = -16$
$2 - 18 = -16$
$-16 = -16 \quad T$

Ex 3.

$$-x + 3y = -3$$
$$-1(2x + 3y = -3) \Rightarrow$$

$$\begin{aligned} -x + \cancel{3y} &= -3 \\ + \quad -2x - \cancel{3y} &= 3 \end{aligned}$$

$$\frac{-3x}{-3} = \frac{0}{-3}$$
$$x = 0$$

$2(0) + 3y = -3$

$\frac{3y}{3} = \frac{-3}{3}$

$y = -1$

$(0, -1)$

Check: $-0 + 3(-1) = -3$
$-3 = -3 \quad T$
$2(0) + 3(-1) = -3$
$-3 = -3 \quad T$

Ex 4.

$$4x + 4y = -8$$
$$-2(2x + 2y = -4) \Rightarrow$$

$$\begin{aligned} \cancel{4x} + \cancel{4y} &= -8 \\ + \quad -\cancel{4x} - \cancel{4y} &= 8 \end{aligned}$$

$0 = 0$ True

Coincidental Lines

$\boxed{\text{Infinite}}$

Ex 5.

$$3(2a + 6z = 4) \Rightarrow \quad \cancel{6a} + 18z = 12$$
$$-2(3a - 7z = 6) \Rightarrow + \quad -\cancel{6a} + 14z = -12$$

$$\frac{32z}{32} = \frac{0}{32}$$
$$z = 0$$

$2a + 6(0) = 4$

$\frac{2a}{2} = \frac{4}{2}$

$a = 2$

$\boxed{(2, 0)}$

Check:
$2(2) + 6(0) = 4$
$4 = 4 \quad T$
$3(2) - 7(0) = 6$
$6 = 6 \quad T$

Ex 6.

$2\ (-7x + 7y = 7) \Rightarrow -\cancel{14x} + \cancel{14y} = 14$

$7\ (2x - 2y = -18) \Rightarrow + \cancel{14x} - \cancel{14y} = -126$

$0 = -112$

False

$\overset{5}{1}8$
$\times 7$
126

Parallel Lines

$\boxed{\text{No solution}}$